Needle models of nonequilibrium growth processes

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Abstract

Non-equilibrium growth processes, such as electrodeposition, dielectric breakdown, viscous fingering, or even bacterial colonies formation, are often driven by instabilities. Accordingly, the resulting growth patterns are usually highly branched fractal structures. In all these processes the growth may be described in terms of a harmonic scalar field Ψ, interpreted for instance as an electrostatic potential or pressure. Additionally, the front is assumed to grow with velocity proportional to the gradient of the field. Such a growth problem is non-linear due to the boundary conditions – the front is unstable under small perturbations. Therefore, even though the basic mechanisms of growth are well understood, the strongly non-linear character of the process makes the latter stages of evolution very complicated, with a strong competition between spontaneously formed dendrite-like structures, and tip-splitting effects when dendrites bifurcate into secondary branches.

In the thesis we considered a simple model of non-equilibrium growth in two spatial dimensions, in which the growth takes place only at the tips of long-and-thin fingers. The quantitative analysis of the model was provided by means of the Loewner equation, which one can use to reduce the problem of the interface motion to that of the evolution of the conformal mapping onto the complex plane. In spite of being considerably simplified, the model allows to describe a strong, nonlinear interaction between the fingers and their competition due to the long-range screening.

In the first part we applied the thin finger model to description of the growth processes in which the envelope of the ramified structure grows in a highly regular manner, with the perturbations smoothed out over the course of time. We showed that the regularity of the envelope growth can be connected to small-scale instabilities leading to the tip splitting of the fingers at the advancing front of the structure. Whenever the growth velocity becomes too large, the finger splits into two branches. In this way it can absorb an increased flux and thus damp the instability. Hence, somewhat counterintuitively, the instability at a small scale results in a stability at a larger scale. In particular we analyzed the growth in a half-plane geometry, in which case the envelopes form perfect semi-circular shapes with non-uniform intensity of the splitting process along the interface. Interestingly, a similar effect can be observed in some 2D combustion experiments.

In the second part we studied patterns formed by viscous fingering in a rectangular network of microfluidic channels. Due to the strong anisotropy of such a system, the emerging patterns have a form of thin needle-like fingers, which interact with each other, competing for an available flow. We developed an upscaled description of this system in which only the fingers are tracked and the effective interactions between them are

introduced, mediated through the evolving pressure field. A complex two-phase flow problem was thus reduced to a much simpler task of tracking evolving shapes in a 2d complex plane. This description, although simplified, turned out to capture all the key features of the system's dynamics and allowed for the effective prediction of the resulting growth patterns.